It may be interesting to establish the coördinates of the terminus of the transition. Since

$$\int_0^\infty \frac{\sin\varphi}{\sqrt{\varphi}}\,d\varphi = \int_0^\infty \frac{\cos\varphi}{\sqrt{\varphi}}\,d\varphi = \sqrt{\frac{\pi}{2}},$$

the coördinates become $x_\infty = y_\infty = \frac{1}{2}\sqrt{\pi/2k}$, $x = y = \frac{1}{2}\sqrt{\pi RL}$.

The coördinates of the transition are related to the Bessel functions as follows:

$$J_{1/2}(\varphi) = \frac{1}{\sqrt{\pi}}\frac{\sin\varphi}{\sqrt{\varphi}}\,;\; J_{-1/2} = \frac{1}{\sqrt{\pi}}\frac{\cos\varphi}{\sqrt{\varphi}}\,;\; C = \tfrac{1}{2}\int_0^\phi J_{-1/2}(\varphi)d\varphi;\; S = \tfrac{1}{2}\int_0^\phi J_{1/2}(\varphi)d\varphi;$$

whence $y = \sqrt{\pi RL}\, S(\varphi)$ and $x = \sqrt{\pi RL}\, C(\varphi)$. (Cf. Jahnke and Emde's Tables, p. 23 seq.)

II. Relating to the Graph of a Cubic Equation having Complex Roots.

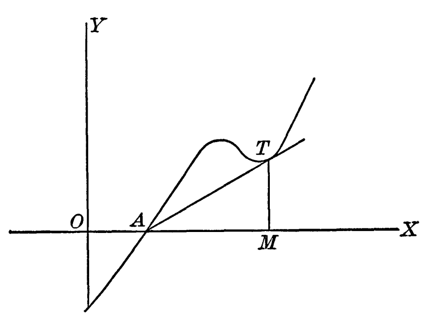
By Edwin S. Crawley, University of Pennsylvania.

The note on "The Graphical Solution of a Cubic Equation having Complex Roots," pp. 70–71 of the Monthly for February, 1918, recalls to my mind something similar which I learned a number of years ago and which might possibly interest some readers of the Monthly.

Every cubic with one real and two imaginary roots is expressible in the form $(x - k)(x^2 - 2px + p^2 + q^2) = 0$, and the graph of

$$y = (x - k)(x^2 - 2px + p^2 + q^2)$$

(*i. e.*, of $y = a_0x^3 + a_1x^2 + a_2x + a_3$) always has a form more or less like the figure. Then it is easy to show that $OM = p$ and $\tan MAT = q^2$, where $p \pm qi$



are the imaginary roots. AT is the tangent to the curve drawn from its real intersection with OX.

For, if $OA = k$ the line $y = \lambda(x - k)$ will be tangent to the curve if

$$x^2 - 2px + p^2 + q^2 - \lambda = 0$$

has equal roots, that is, if $\lambda = q^2$; and $y = q^2(x - k)$ touches the curve at $x = p$.

[Since the foregoing was put in type the editors have called my attention to the fact that the same construction was given by Irwin and Wright in the *Annals of Mathematics,* Vol. 19 (1917), pp. 157–158.]

III. Relating to the Selection of Material for Class Reviews.

By G. R. Clements, United States Naval Academy.

I would like to suggest as a topic for discussion in the Monthly the best use that can be made of the time available for reviews in courses in mathematics.

It seems to me that the best test of a student's mental capacity and of his mastery of a particular course of study in mathematics is his ability to take a pertinent problem and analyze it to see what principles are involved, to select for its solution the most suitable tools from those he has been accumulating during his term's work and to use them intelligently to derive and discuss the results that must follow from his data. And I know of no better way to help a student to correlate his material and get a comprehensive grasp of it than to set him to work at the end of a course on a list of problems carefully selected but *not* closely graded.

When I was teaching at the University of Wisconsin, we secured better results than we had previously obtained in the course in mathematical theory of investments by going rather quickly through the course and then spending a considerable amount of time on just such a list of problems, where the student could have no idea in advance as to whether the solution of a particular problem involved the principles of the first chapter of his text, or of the last chapter, or of both. I have followed a somewhat similar course in analytic geometry, assigning twice each term a list of problems from outside the text (and regarded as somewhat difficult by the students), solutions to be handed in at the end of two weeks, the daily work being somewhat lightened in the meantime.

The increasing list of separate problem texts and the number of texts that have considerable lists of general problems as an appendix would seem to make such a plan rather easy of introduction so far as the mere selection of material is concerned. For example, Miller and Lilly's *Analytic Mechanics* has a very considerable list of miscellaneous problems, and I can think of no better way of reviewing the subject, if that book is being used as a text, than to set the students to mastering a selected portion of this list.

I believe the criticisms that we ourselves and our colleagues in allied departments make of the results of our teaching would be considerably softened if we could conclude each of our courses by going over its subject matter a last time in some such manner as I have suggested.